Voltage/Current Relations in Phasor Form

We begin with the First Order Telegrapher's Equations:

$$\frac{\partial V(z,t)}{\partial z} = -\left(R\,I(z,t) + L\,\frac{\partial I(z,t)}{\partial t}\right),$$
$$\frac{\partial I(z,t)}{\partial z} = -\left(G\,V(z,t) + C\,\frac{\partial V(z,t)}{\partial t}\right)$$

Substituting the Phasor Forms of *I* and *V* gives:

$$\frac{\partial}{\partial z}\left(V_s e^{j\omega t}\right) = -\left(R\,I_s e^{j\omega t} + L\,j\omega I_s e^{j\omega t}\right) \implies \frac{dV_s}{dz} = -\left(R + j\omega L\right) I_s,$$

$$\frac{\partial}{\partial z}\left(I_s e^{j\omega t}\right) = -\left(G\,V_s e^{j\omega t} + C\,j\omega V_s e^{j\omega t}\right) \implies \frac{dI_s}{dz} = -\left(G + j\omega C\right) V_s.$$

We can then Define the Per-Unit Series Impedance and Shunt Admittance: $Z \equiv R + j\omega L \quad Y \equiv G + j\omega C$

$$\frac{dV_s}{dz} = -Z\,I_s, \qquad \frac{dI_s}{dz} = -Y\,V_s$$

Complex Impedance

Suggested Readings for Transmission line
Hayt, Chapter 10
Fawwaz: Chapters 1, 2

We begin with the Wave Solutions for Voltage and Current:

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z},$$
$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z},$$

Substituting $V_s(z)$ into $\frac{dV_s}{dz} = -ZI_s$

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -Z\left(I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}\right)$$